INTEGERS 3
STUDENT PACKET

INTEGER MULTIPLICATION AND DIVISION

Parent (or Guardian) signature ____________________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 3.5.) Key mathematical vocabulary is underlined throughout the packet.

distributive property	exponential notation
factor	inverse operation
product	quotient

MORE OF MR. MORTIMER'S MAGIC

Merrimack Mortimer is at it again. He decides that he wants to heat up and cool down his liquids faster by placing in and removing pre-made packages of magic cubes. Remember, each cube changes the temperature by 1 degree.

Explain how the temperature of the liquid changes in each of the following situations. Remember that each situation is totally independent.

1. Mortimer puts in 2 packs of 4 hot cubes.

2. Mortimer puts in 5 packs of 4 cold cubes.

3. Mortimer takes out 4 packs of 3 hot cubes.

4. Mortimer takes out 3 packs of 5 cold cubes.

5. Describe four different ways for Mortimer to make a liquid 24 degrees hotter using pre-made packs.

Mortimer puts in:	Mortimer puts in:
Mortimer removes:	Mortimer removes:

MULTIPLYING INTEGERS

We will use a counter model to generalize rules for integer multiplication.

GETTING STARTED

Compute.

1. 15 + (-10)	2. 5 + (-10)	3. 0 + (-10)
4. 15 – (-10)	5. 5 – (-10)	6. 0 – (-10)

Several sets of zero pairs are provided. Make a drawing that illustrates each instruction below, and write the result.

7. Put in 2 groups of 5 ***positive*** counters. What is the value now? + + + + + – – – – – + + + + + – – – – –	8. Put in 2 groups of 5 ***negative*** counters. What is the value now? + + + + + – – – – – + + + + + – – – – –
9. Take out 2 groups of 5 ***positive*** counters. What is the value now? + + + + + – – – – – + + + + + – – – – –	10. Take out 2 groups of 5 ***negative*** counters. What is the value now? + + + + + – – – – – + + + + + – – – – –

MULTIPLYING INTEGERS WITH COUNTERS 1

1. Look up factor and product in section 3.5 and record them in My Word Bank.

Use these sentence frames to help you think through integer multiplication.
Do not write in these.

- Begin with a workspace that has a value equal to 0.
- The first factor is positive. We will place _____ groups on the workspace.
- The second factor is __________.
  positive/negative
- Each group will contain _____ __________ counter(s).
  positive/negative
- The result is _____ __________ counter(s).
  positive/negative

Follow your teacher's directions to complete problems 2 and 3.

2.	3.

Compute each product. Record using positive symbols (+) and negative symbols (–).

4. (2) • (4)	5. (2) • (-4)
6. (3) • (2)	7. (3) • (-2)

Refer to the problems above to complete problems 8 and 9.

8. The product of a positive number and a positive number is a ______________ number.

9. The product of a positive number and a negative number is a ______________ number.

MULTIPLYING INTEGERS WITH COUNTERS 2

Use these sentence frames to help you think through integer multiplication.
Do not write in these.

- Begin with a workspace that has a value equal to 0.
- The first factor is negative. We will remove _____ groups on the workspace.
- The second factor is __________.
  positive/negative
- Each group will contain _____ __________ counter(s).
  positive/negative
- Introduce _____ zero pairs to remove these groups.
- The result is _____ __________ counter(s).
  positive/negative

Follow your teacher's directions to complete problems 1 and 2.

1.	2.

Compute each product. Record using positive symbols (+) and negative symbols (–).

3. (-2) • (4)	4. (-2) • (-4)
5. (-3) • (2)	6. (-3) • (-2)

Refer to the problems above to complete problems 7 and 8.

7. The product of a negative number and a positive number is a ______________ number.

8. The product of a negative number and a negative number is a ______________ number.

PRACTICE 1

Compute. Refer to the script from the previous pages and draw pictures as desired.

1. (4) • (-5)	2. (-4) • (3)	3. (-3) • (-5)
4. (-5) • (2)	5. (3) • (-1)	6. (-1) • (-2)

7. Summarize the rules for integer multiplication.

The product of two positive numbers is _____________.

The product of two negative numbers is _____________.

The product of one positive and one negative number is _____________.

Compute without using counters or drawing pictures. If NOT done mentally, show your work.

8. (-3) • (-10)	9. (3) • (-10)	10. (-3) • (10)
11. (-30) • (-10)	12. (-3) • (100)	13. (30) • (-100)
14. (-3) • (17)	15. (-3) • (-241)	16. (-31) • (25)
17. -3 + (-10)	18. 3 + (-10)	19. -3 + 10

DIVIDING INTEGERS

We will use patterns to develop the integer division rules.

GETTING STARTED

1. Look up <u>quotient</u> and <u>inverse operation</u> at the end of the packet and record them in My Word Bank.

2. Use the fact that division is the inverse of multiplication to fill in the blanks.

a. (3) • (7) = 21	(21) ÷ (7) = _____
	(21) ÷ (_____) = 7
b. (5) • (_____) = 30	(_____) ÷ (5) = _____
	(_____) ÷ (_____) = 5
c. (_____) • (9) = 18	(_____) ÷ (9) = _____
	(_____) ÷ (_____) = 9

3. Choose any two positive numbers, and write the multiplication statement showing their product.

_____ • _____ = _____

4. Write the two division statements related to the multiplication statement you created for problem 3.

_____ ÷ _____ = _____ and _____ ÷ _____ = _____

RELATING MULTIPLICATION AND DIVISION

1. Use the fact that division is the inverse of multiplication to fill in the blanks.

a.	(4) • (______) = -12	(-12) ÷ (4) = ______ (-12) ÷ (______) = 4
b.	(______) • (2) = -8	(______) ÷ (2) = ______ (______) ÷ (______) = 2
c.	(-3) • (______) = 6	(______) ÷ (-3) = ______ (______) ÷ (______) = -3

We will use the shorthand **pos** for a positive number and **neg** for a negative number. Circle the correct result.

2. pos ÷ pos → pos neg	3. neg ÷ neg → pos neg
4. pos ÷ neg → pos neg	5. neg ÷ pos → pos neg

6. How do the rules for multiplying integers compare to the rules for dividing integers?

Compute.

7. -14 ÷ 7	8. 15 ÷ (-3)	9. -25 ÷ (-5)
10. $\frac{-20}{-4}$	11. $\frac{24}{-6}$	12. $\frac{-170}{10}$

PRACTICE 2

Compute.

1. (-3) • 7	2. (-27) ÷ 3	3. $\frac{-12}{4}$
4. (-5)(-9)	5. (-40) ÷ (-10)	6. $\frac{-26}{-13}$
7. 8(-4)	8. 56 ÷ (-7)	9. $\frac{12}{-4}$
10. (-4)(-40)	11. (-200) ÷ (-50)	12. $\frac{-77}{-11}$
13. (-600)7	14. -320 ÷ 80	15. $\frac{-560}{-70}$
16. 5 – 10	17. -5 – 10	18. -5 – (-10)

19. Silvia hid some counters in her left hand and some more in her right hand. Each hand had either all negatives or all positives. She challenged her group to answer each question. Clearly explain how her group should respond.

a. "The product of the amounts in my hands is 50, and the sum is -15. What do I have in each hand?"

b. "The product of the amounts in my hands is -36, and the sum is 9. What do I have in each hand?"

DETERMINING THE SIGN OF A PRODUCT OR QUOTIENT

Compute each product.

1. (-1) • (-2) • (3)	2. (-1) • (-2) • (-3)	3. (-1) • (-2) • (3) • (-4)
4. (-1) • (-2) • (-3) • (-4)	5. (-1) • (2) • (-3) • (4)	6. (-1) • (0) • (-3) • (-4)

7. Make conjectures about multiplying **nonzero** numbers.
   a. If there are an odd number of negative factors, the product is ______________.
      positive / negative
   b. If there are an even number of negative factors, the product is ______________.
      positive / negative

Compute each quotient.

8. $\frac{-10}{2}$	9. $\frac{10}{-2}$	10. $\frac{-10}{-2}$
11. $-\frac{10}{2}$	12. $-\left(\frac{10}{2}\right)$	13. $-\left(-\frac{10}{2}\right)$

Write <, >, or = for each.

14. (-1) • (-1) • (1) _____ (-1) • (-1) • (-1)	
15. (-2) • (-3) • (-4) • (10) _____ -2 • (-3) • (-4) • (-10)	
16. 6(-5)(-2) _____ (-6)(5)(2)	
17. (-2) • (-3) • (-4) • (10) _____ -2 • (3) • (-4) • (-10)	
18. $\frac{-40}{10}$ _____ $\frac{-40}{-10}$	19. $\frac{-36}{-12}$ _____ $-\left(-\frac{36}{12}\right)$
20. -4 + (-8) _____ -4 – (-8)	21. -2 – 6 _____ -2 + (-6)

DIVISION INVOLVING ZERO

	Statement/Question	Division Expression	Does the question make sense mathematically? What is the answer?
1.	Four friends are equally sharing 16 grapes. How many grapes does each friend get?	$\frac{\square}{\square}$	
2.	Four friends are equally sharing 0 grapes. How many grapes does each friend get?	$\frac{\square}{\square}$	
3.	Four friends are equally sharing 2 strawberries. How many strawberries does each friend get?	$\frac{\square}{\square}$	
4.	Zero friends are equally sharing 15 strawberries. How many strawberries does each friend get?	$\frac{\square}{\square}$	

Mathematically, we say that division *by* zero is **undefined**.

Fill in each box with a solution if one exists, an **N** if there is no solution, or an **I** if an infinite number of solutions exist.

5a. $2 \bullet \square = 8 \rightarrow \frac{8}{2} = \square$	5b. $0 \bullet \square = 8 \rightarrow \frac{8}{0} = \square$
6a. $10 \bullet \square = 0 \rightarrow \frac{0}{10} = \square$	6b. $0 \bullet \square = 0 \rightarrow \frac{0}{0} = \square$

PRACTICE 3

Compute, if possible.

1. -20 • (-30) • (-200)	2. -80 ÷ 10	3. $-\frac{45}{9}$
4. (-10)(-20)(30)	5. 64 ÷ (-8)	6. $-\left(\frac{36}{6}\right)$
7. (-1)(-2)(-3)(-4)(-5)	8. -60 ÷ (-30)	9. $-\left(-\frac{28}{7}\right)$
10. (-12)(0)(-13)(210)	11. 0 ÷ 10	12. 20 ÷ 0
13. (-17)(53)(0)(-27)	14. $\frac{0}{3}$	15. $\frac{3}{0}$
16. -120 + 20	17. -80 + (-40)	18. -30 + 70
19. 100 – (-200)	20. -100 – (-200)	21. 100 – 200

22. Why is $\frac{-10}{5}$ not equal to $\frac{-10}{-5}$?

23. If the product of six integers is negative, at most how many of the integers can be negative?

24. Lydia hid some counters in each hand. Each hand had either all negatives or all positives. Lydia said to her group, "The sum of the amounts in my hands is -12 and the product is -28. What do I have in each hand?" How should her group respond?"

ORDER OF OPERATIONS

We will make sense of the order of operations conventions and solve problems involving integers.

GETTING STARTED

Put the following statements in an order you think makes the most sense.
Then predict whether you think most of your classmates will agree with you or not.

1.
_____ Tie your shoelaces.

_____ Put on your socks.

_____ Put on your shoes.

Prediction:

2.
_____ Eat dinner.

_____ Do homework.

_____ Do something recreational like playing basketball or drawing a picture.

Prediction:

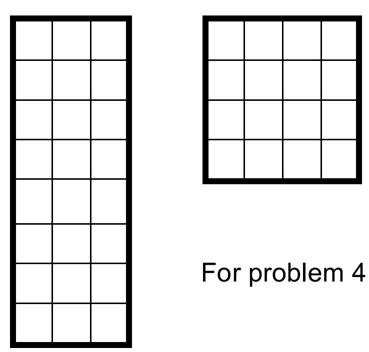
You do not need to calculate anything for the following problems. Place operation symbols in the boxes to make numerical expressions that are correct translations of the situation.

3. The cost of buying 2 bottles of juice for $1.50 each and 3 bags of pretzels for $2.00 each.

2 ☐ 1.5 ☐ 3 ☐ 2

4. The total area of the two rectangles combined.

3 ☐ 8 ☐ 4 ☐ 4



For problem 4

EXPONENTS

1. Look up exponential notation in section 3.5 and record it in My Word Bank.

Write each expression as an appropriate product. Then compute.

2. 5^2 = _____ • _____ = __________

3. 3^4 = ___________________________________ = __________

4. 2^6 = ___________________________________ = __________

5. 17^1 = ___________________________________ = __________

6. When computing an expression, Horton wrote $2^3 = 6$. Explain Horton's mistaken thinking.

Compute.

7. 3^2	8. 2^5	9. 4^3
10. 6^2	11. $3^2 + 3^4$	12. $2^3 \bullet 3^2$

Write each of the following as a base with an exponent

13. $2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2$	14. $6 \bullet 6$	15. $0 \bullet 0 \bullet 0$

THE ORDER OF OPERATIONS CONVENTIONS

Follow your teacher's directions to complete problems 1 – 7.

1.	2.
3.	4.

Order of Operations

5.
   A.

   B.

   C.

   D.

6. Example 1	7. Example 2

8. Simplify the expression: $\dfrac{9}{1-4} + \dfrac{-20+3\bullet 4}{10-(1+5)}$

PRACTICE 4

Compute.

1. $\dfrac{(-6)(-2)-2}{-10}$	2. $\dfrac{-1+(-1)+4^2}{2}$
3. $\dfrac{-6-2}{2-(-2)}$	4. $\dfrac{2-(-2)}{-6-2}$
5. $20 + (-8 + 6) - 3^2 \bullet 2$	6. $20 + -8 + (6 - 3^2) \bullet 2$
7. $10 - 14 \div 7 - 3 \bullet 4$	8. $10 - (14 \div 7 - 3 \bullet 4)$

Use all four of the numbers 2, 3, 4, and 5 exactly once in each problem below. Use any of the four operations and any grouping symbols as needed.

9. Write an expression that is equal to 1.	10. Write an expression that is equal to 6.

INTEGER PROBLEMS

1. A fish is swimming 15 feet below sea level.

   a. If zero represents sea level, what number represents the fish's elevation? _______

   b. A dolphin is swimming 3 times as deep as the fish. What numerical expression represents the elevation that is 3 times the depth of the fish? ___________

   c. What number represents the elevation of the dolphin? _______

2. During a cold week in Wisconsin, the temperature each day at noon in Fahrenheit was 4°, -6°, -1°, 3°, and 0°.

   a. What numerical expression can be used to find the average noontime temperature for the week?

   b. What was the average temperature at noon for the week? _______

3. During the same cold week in Wisconsin, the temperature each day at midnight in Fahrenheit was -4°, -6°, -10°, -3°, and -7°.

   a. What numerical expression can be used for the average midnight temperature for the week?

   b. What was the average temperature at midnight for the week? _______

4. The water level in a lake rose 5 inches per month for 3 months and then dropped 7 inches per month for 4 months.

   a. What numerical expression can be used to describe the elevation change in inches?

   b. After 7 months, was the water level of the lake higher or lower than the starting water level? _________________ By how much? ____________________

   c. Write this gain or loss as a number. _______

REVIEW

TARGET PRACTICE

Your teacher will turn over 4 integer cards.

Record their values: _______ _______ _______ _______

For each problem below, write an expression using the four numbers above exactly once each. Show your work. You may use any of the four operations and any grouping symbols you know.

1. Write an expression with a value as close to 1 as possible. Expression: ______________________	2. Write an expression with a value as close to -1 as possible. Expression: ______________________
3. Write an expression with the greatest value possible. Expression: ______________________	4. Write an expression with the least value possible. Expression: ______________________

POSTER PROBLEM: NUMBER TRICKS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _________.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

Round 1:

Poster #	1	2	3	4	5	6	7	8
Start #	1	2	3	4	5	6	7	8

A. Create a 4-column chart like this to use for steps 1-7:

step number	step directions	Round 1 work	Round 2 work

Do Step 1: Copy your start number from the table above onto your chart.
Do Step 2: Multiply the start number by -4.

B. Do Step 3: Add -10 to the result.
Do Step 4: Subtract -6 from the result.

C. Do Step 5: Divide the result by -4.
Do Step 6: Subtract the given start number from the result.

D. Do Step 7: Add -1 to this result. Circle this number.

(For **Round 2**, change roles, start with the opposite of your start number, and repeat. For example, if you started with 3 before, now start with -3.)

Part 3: Return to your seats. Work with your group.

1. Was the circled number on every poster the same? _____
2. If not, use the start number given to you by your teacher and rework the problem.

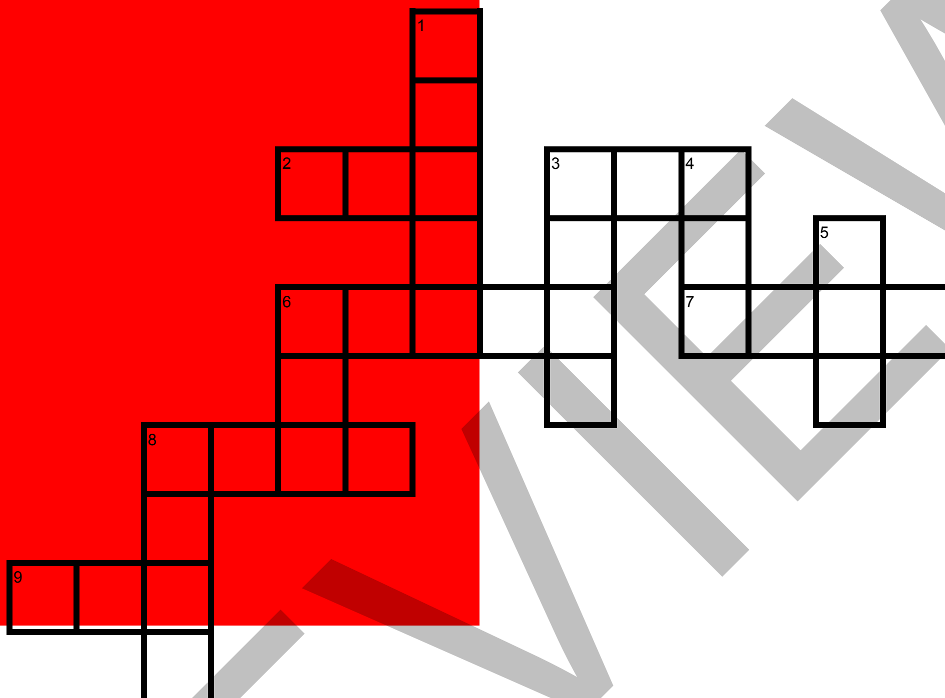
ORDER OF OPERATIONS PAIR SHARE

Partner A

- Do "across" problems on other paper.
- Check B's work on the "down" problems using a calculator.

Partner B

- Do "down" problems on other paper.
- Check A's work on the "across" problems using a calculator.



Across

2. $\left(\dfrac{-8-2}{-4-6}\right)+379$

3. $\left(\dfrac{12-18}{-3}\right)+245$

6. $\left(\dfrac{-3-4\cdot 2-5}{1-2}\right)\bullet(1{,}000)$

7. $-(-600-800-200)$

8. $\left(15-\dfrac{21}{3}+12\right)\bullet(80)$

9. $-4-(-3)(-6)+22+150$

Down

1. $[100+(-2)(-2)(-2)]\bullet(1{,}000)$

3. $[-3-5(-6)]\cdot(100)$

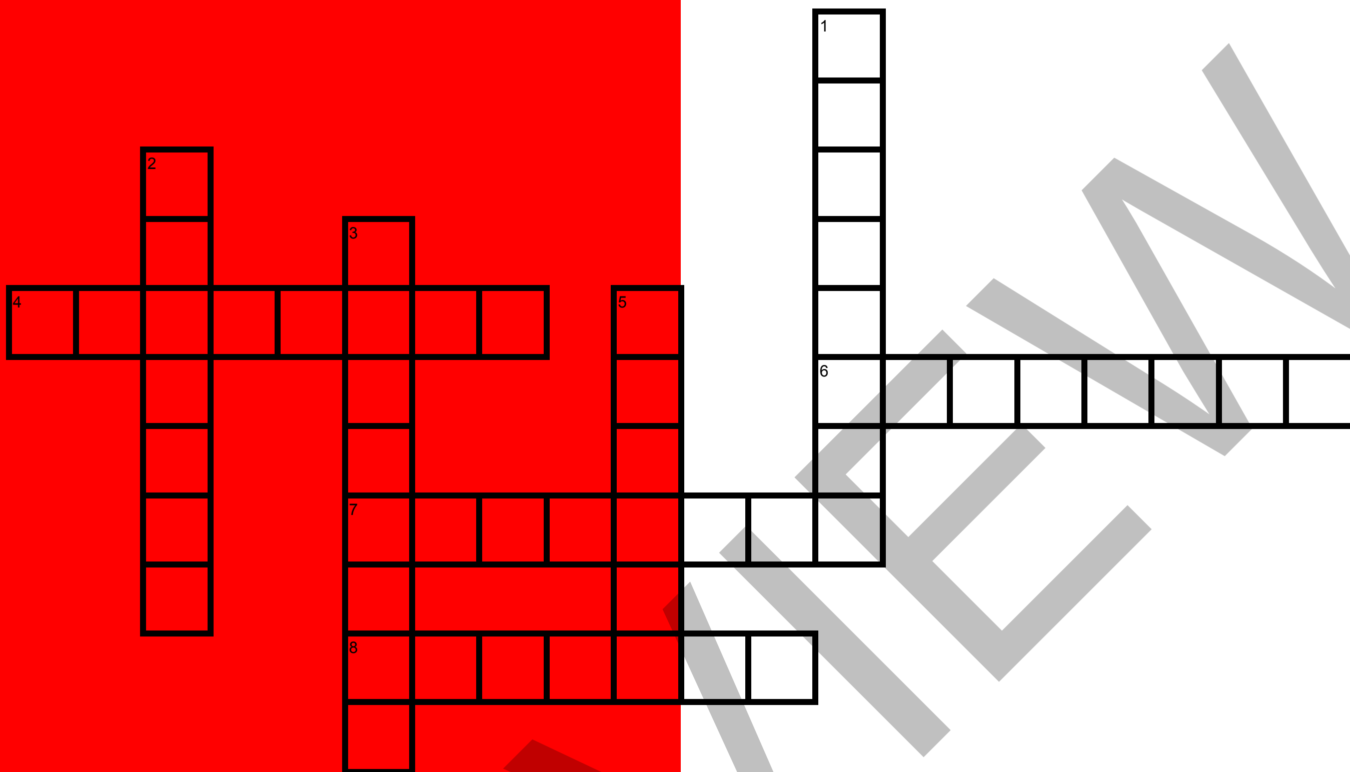
4. $\left(\dfrac{-8+4-6}{-11+1}\right)+720$

5. $\left(\dfrac{-5-3\cdot 5}{-7-3}\right)\bullet(200)$

6. $-4+\left(\dfrac{-16}{2}\right)(-3-1)+72$

8. $100(24-2\bullet 3)$

VOCABULARY REVIEW



Across

4 The result of a division problem.

6 _____ are the numbers 0, 1, 2, 3, and their opposites.

7 The product of a negative number and a negative number is a _____ number.

8 Multiplication and division are _____ operations. Addition and subtraction have this same relationship too.

Down

1 The quotient of a negative number and a positive number is a _____ number.

2 The _____ is the result of multiplying two or more numbers.

3 The combination of one positive counter and one negative counter is a _____ _____ (2 words).

5 In the equation $7 \bullet 8 = 56$, one _____ is 7 and the other is 8.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES

Word or Phrase	Definition
distributive property	The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers a, b, and c. $3(4 + 5) = 3(4) + 3(5)$ and $(4 + 5)8 = 4(8) + 5(8)$
division	Division is the mathematical operation that is inverse to multiplication. For $b \neq 0$, division by b is multiplication by the multiplicative inverse $\frac{1}{b}$ of b, $a \div b = a \bullet \frac{1}{b}$. Other notations for $a \div b$ are $\frac{a}{b}$, a/b, and $b\overline{)a}$. "Twelve divided by 2" may be written $12 \div 2$, $\frac{12}{2}$, or $2\overline{)12}$. Also, $12 \div 2$ is equivalent to $12 \bullet \frac{1}{2} = 6$.
exponential notation	The exponential notation b^n (read as " b to the power n ") is used to express n factors of b. The number b is the base, and the number n is the exponent. $2^3 = 2 \cdot 2 \cdot 2 = 8$. The base is 2 and the exponent is 3. $3^2 = 3 \cdot 3 = 9$. The base is 3 and the exponent is 2.
factor	In a multiplication problem, a factor is a number or expression being multiplied. See product. 3 • 5 = 15 factor factor product
inverse operation	The inverse operation to a mathematical operation reverses the effect of the operation. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.
product	A product is the result of multiplying two or more numbers or expressions. The numbers or expressions being multiplied to form the product are factors of the product. The product of 7 and 8 is 56, written $7 \bullet 8 = 56$. The numbers 7 and 8 are both factors of 56.
quotient	In a division problem, the quotient is the result of the division. See division. In $12 \div 3 = 4$, the quotient is 4.

Symbols for Multiplication

The product of 8 and 4 can be written as:

8 times 4	8×4	$8 \bullet 4$	(8)(4)	$\begin{array}{r} 8 \\ \times\ 4 \\ \hline \end{array}$

The product of 8 and the variable x is written simply as $8x$. We generally avoid using the symbol $\times$ for multiplication because it could be misinterpreted as the variable x. Also, we are cautious about the use of the symbol $\bullet$ for multiplication because it could be misinterpreted as a decimal point.

Symbols for Division

The quotient of 8 and 4 can be written as:

8 divided by 4	$8 \div 4$	$4\overline{)8}$	$\frac{8}{4}$	8/4

In algebra, the preferred way to show division is with fraction notation.

Mr. Mortimer's Magic Hot and Cold Cubes for Multiplication

Mr. Mortimer invented hot cubes and cold cubes that never change temperature. For example, ice cubes melt. Magic cold cubes do not. Place 1 cold cube in a liquid and it decreases the temperature by 1 degree. Place a hot cube in a liquid and it increases the temperature by 1 degree.

Here is how to visualize multiplication with Mr. Mortimer's cubes.

Hot Cubes:

- If you put in packs of hot cubes to a liquid, the liquid heats up.
  For example, adding 2 packs of 10 hot cubes is like adding $2 \bullet 10 = 20$ hot cubes.
  The liquid heats up by 20 degrees.

- If you take out packs of hot cubes from a liquid, the liquid cools down.
  For example, subtracting 2 packs of 10 hot cubes is like subtracting $2 \bullet 10 = 20$ hot cubes.
  The liquid cools down by 20 degrees.

Cold Cubes:

- If you put in packs of cold cubes to a liquid, the liquid cools down.
  For example, adding 2 packs of 10 cold cubes is like adding $2 \bullet 10 = 20$ cold cubes.
  The liquid cools down by 20 degrees.

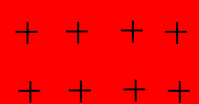
- If you take out packs of cold cubes from a liquid, the liquid heats up.
  For example, subtracting 2 packs of 10 cold cubes is like subtracting $2 \bullet 10 = 20$ cold cubes.
  The liquid heats up by 20 degrees.

Counter Multiplication Sentence Frames

- Begin with a workspace that has a value equal to 0.
- **If the first factor is positive**, we will place ______ groups on the workspace.
  If the first factor is negative, we will remove ______ groups on the workspace.
- The second factor is ___________.
  (positive/negative)
- Each group will contain ______ ___________ counter(s).
  (positive/negative)
- Introduce ______ zero pairs to remove these groups (if needed).
- The result is ______ ___________ counter(s).
  (positive/negative)

Integer Multiplication Using Counters

2(4) = 8

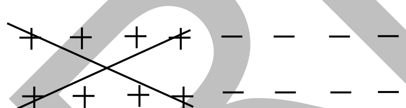


- Start with a work space equal to zero.
- The first factor is positive.
  We will put 2 groups on the workspace.
- The second factor is positive.
  Each group will contain 4 positive counters.
- [No zero pairs needed.]
- The result is 8 positive counters.

2(-4) = -8

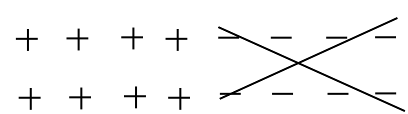
- Start with a work space equal to zero.
- The first factor is positive.
  We will put 2 groups on the workspace.
- The second factor is negative.
  Each group will contain 4 negative counters.
- [No zero pairs needed.]
- The result is 8 negative counters

-2(4) = -8



- Start with a work space equal to zero.
- The first factor is negative.
  We will remove 2 groups from the workspace.
- The second factor is positive. Each group will contain 4 positive counters.
- Introduce at least 8 zero pairs.
- The result is 8 negative counters.

-2(-4) = 8



- Start with a work space equal to zero.
- The first factor is negative.
  We will remove 2 groups from the workspace.
- The second factor is negative. Each group will contain 4 negative counters.
- Introduce at least 8 zero pairs.
- The result is 8 positive counters.

Rules for Multiplication of Integers

Rule 1: The product of two numbers with the same sign is a positive number.

Think: (+)(+) = (+) and (–)(–) = (+)

Rule 2: The product of two numbers with opposite signs is a negative number.

Think: (+)(–) = (–) and (–)(+) = (–)

Rules for Division of Integers

Rule 1: The quotient of two numbers with the same sign is a positive number.

Think: $\frac{(+)}{(+)} = (+)$ and $\frac{(-)}{(-)} = (+)$

Rule 2: The quotient of two numbers with opposite signs is a negative number.

Think: $\frac{(+)}{(-)} = (-)$ and $\frac{(-)}{(+)} = (-)$

Mathematical Separators

Parentheses ( ) and square brackets [ ] are used in mathematical language as separators. The expression inside the parentheses or brackets is considered as a single unit. Operations are performed inside the parentheses before the expression inside the parentheses is combined with anything outside the parentheses.

$5 - (2 + 1) = 5 - (3) = 2$

In the example below, operate on the expression in the innermost separator first and work your way out.

$20 \div [6 - (4 - 8)] = 20 \div [6 - (-4)] = 20 \div 10 = 2$

The horizontal line used for a division problem is also a separator. It separates the expressions above and below the line, so the numerator and denominator must be simplified completely before dividing.

$$\frac{20 + 10}{5 \bullet 2} = \frac{30}{10} = 3$$

Order of Operations

There are many mathematical conventions that enable us to interpret mathematical notation and to communicate efficiently about common situations. The agreed-upon rules for interpreting mathematical notation, important for simplifying arithmetic and algebraic expressions, are called the order of operations.

1. Do the operations in grouping symbols first (e.g., use rules 2-4 inside parentheses).
2. Calculate all the expressions with exponents.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

$$\frac{11+(17-2\bullet 3^2)}{5} = \frac{11+(17-2\bullet 9)}{5} = \frac{11+(17-18)}{5} = \frac{11+(-1)}{5} = \frac{10}{5} = 2$$

There are many times for which these rules make complete sense and are quite natural. Take this case, for example:

You purchase 2 bottles of water for $1.50 each and 3 bags of peanuts for $1.25 each. Write an expression for this situation, and simplify the expression to find the total cost.

$$\underbrace{2 \bullet (1.50)}_{3.00} + \underbrace{3 \bullet (1.25)}_{3.75} = \$6.75$$

In this problem, it is natural to find the cost of the 2 bottles of water and then the cost of the 3 bags of peanuts prior to adding these amounts together. In other words, we perform the multiplication operations before the addition operation.

Note however that if we were to perform the operations in order from left to right (as we read the English language from left to right), we would obtain a different result:

WRONG → 2(1.50) = 3 → 3 + 3 = 6 → 6(1.25) = $7.50

Using Order of Operations to Simplify Expressions		
Order of Operations	**Example** $\frac{40 - 2 \bullet 5^2 - (8 - 6)}{4 + 2 \bullet 10}$	**Comments**
Simplify expressions within grouping symbols.	$\frac{40 - 2 \bullet 5^2 - 2}{4 + 2 \bullet 10}$	Parentheses are grouping symbols: $(8 - 6) = 2$ The fraction bar, used for division, is also a grouping symbol, so the numerator and denominator must be simplified completely prior to dividing.
Calculate all the expressions with exponents.	$\frac{40 - 2 \bullet 25 - 2}{4 + 2 \bullet 10}$	$5^2 = 5 \bullet 5 = 25$
Perform multiplication and division from left to right.	$\frac{40 - 50 - 2}{4 + 20}$	In the numerator: Multiply $2 \bullet 25 = 50$. In the denominator: Multiply $2 \bullet 10 = 20$.
Perform addition and subtraction from left to right.	$\frac{-12}{24}$	In the numerator: Subtract from left to right $40 - 50 - 2 = -12$. In the denominator: Add $4 + 20 = 24$
	$\frac{-1}{2}$ or $-\frac{1}{2}$	Now the groupings in both the numerator and denominator have been simplified, so the final division can be performed.

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